Comparative Study Between PI and Sliding Mode Controllers for Flexible Joint Manipulator Driving by Brushless DC Motor

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***Abstract*—In this paper, a comparison between PI and sliding mode control is proposed for the control for flexible joint manipulator driving by Brushless DC motor (BDCM)to improve the performance in the presence of errors in the parameters identification. First, the model of flexible joint manipulator was presented. Then, a PI control is studied and designed for tracking problem of the single flexible joint manipulator. Besides, sliding mode control is studied and designed also for the same problem. Finally, those strategies of control are implemented in the Matlab-Simulink environment to show the high performances and a comparative study of the different strategies of PI and sliding mode controllers was achieved. Although the presence of flexibility in the joint, the simulation results show the high performances of sliding mode controller by converging the error to zero and guaranteeing stability of the robot.**

***Keywords*—Control, PI controller, Sliding Mode , Flexible Joint Manipulator, Brushless DC Motor(BDCM).**

I. INTRODUCTION

Brushless DC motors(BDCM) have achieved a big breakthrough along the last decades in multiple applications such as aeronautics, food and chemical industries, electric vehicles, medical instruments, computer peripherals and especially robotics. Thanks to his control simplicity, the BDCM is considered as the most suitable actuator for a robot manipulator. Putting into consideration that a fast and precise movements are necessary to provide to the robot manipulator an excellent dynamic response, it has been shown necessary to require a high dynamic performance of the motor providing in this case a high electromagnetic torque. For this reason, we can not discarded the use of a transmission gearbox system [1]. Similarly, the selection of the control system depend on the characteristics of the motor, as well as the reduction ratio of the gearbox system. The most common control reported in the literature regarding the association of the robot equipped with a brushless motor and high torque gearbox combination is based on tracking trajectories. It is based on the generation of the trajectories which describe the evolution in time of each joint. These trajectories must comply with specific constraints such as joint limits of the robot, the feasibility with respect to engine power, etc..Also, joint flexibility represents one of the most important approaches to enhance the performance of the control. This latter has received a conscientious attention thanks to its high speed, low cost and weight, small size and wide number of applications [2], [3], [4]. However, the complexity of model and control increases. So, a robust strategy of control is required. For this reason, we have opted the PI control representing an essential classical linear control methods developed for flexible robotic manipulators to benefit from its design simplicity and robustness [5]. In other hand, we have opted the sliding mode control which has proved its effectiveness through reported theoretical studies particularly in the areas of robotics and motors. Thanks to its insensitivity to the variation of internal and external parameters and its adaptation to little known system due to the identification problems in system parameters and model simplification, the sliding mode control represents one of the most efficient in variable structure control [6], [7].

To evaluate the performance of the control strategy algorithm based on PI and sliding mode controllers for flexible joint manipulator driving by Brushless DC motor in this paper, we should go through the simulation. In the first part, we will describe the model of our system. In the second part, the strategy of PI controller will be designed. In the third part, the strategy of sliding mode controller will be treated by synthesizing the surface sliding and giving the control law. Finally and in the fourth part, we will implement the strategy of control previously described for PI and sliding mode controllers in the matlab-simulink environment and a comparison between them will be done.

II. MODEL OF THE SYSTEM

A. Model of Flexible Joint Manipulator

A flexible element may be represented by multiple rigid elements connected by springs such as shown in the figure 1.

Referring to *Lagrange* equation, the model of n flexible joints robot can be expressed as follows [4], [6], [8]:

$$A(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + K(q - \frac{\theta}{\eta N}) = 0 \quad (1)$$

$$J\ddot{\theta} - K(q - \frac{\theta}{\eta N}) = \Gamma \quad (2)$$

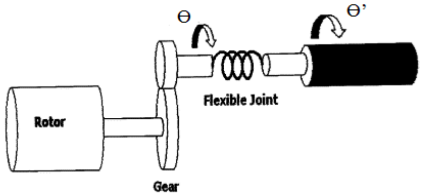

Fig. 1. Manipulator with flexible joint

where:

- q is the n link angular position vector,
- $\dot{q}$ is the n link angular speed vector,
- $\ddot{q}$ is the n link angular acceleration vector,
- A(q) is the nxn positive inertia symmetric matrix,
- $C(q,\dot{q})\dot{q}$ represents the centrifugal and *Coriolis* forces matrix,
- G(q) represents the gravity torque vector,
- J is the matrix inertia,
- K represents joint stiffness,
- θ is the motor angle vector,
- Γ is the torque vector applied to the n axes of the robot,

In the following, we consider that: $q = q_1$, $\dot{q} = \dot{q_1}$, $\ddot{q} = \ddot{q_1}$, $\theta = \theta_1$ and $\Gamma = \Gamma_1$ represent the different parameters previously defined for a single flexible joint.

For single flexible joint, the model of manipulator can be expressed as follows [3]:

$$J_1\ddot{q} + m_1 g l_1\ sin(q)\ +\ K(q - \frac{\theta}{\eta\ N})\ =\ 0 \tag{3}$$

$$J_m\ddot{\theta}\ -\ K(q\ -\ \frac{\theta}{\eta\ N})\ =\ \Gamma \tag{4}$$

where:

- J_1 is the link inertia of the first articulation,
- J_m is the motor inertia of the first articulation,
- m_1 is the mass of the first articulation,
- l_1 is the length of the first articulation,

where N is the reduction report , η is the efficiency of the gearbox and the gravity constant g is equal to $9.81\ m.s^{-2}$.

B. Model of Brushless DC Motor(BDCM)

The Brushless DC motor can be written by the following equations:

$$\frac{d\Omega_m}{dt} = -\frac{f}{J_m}\Omega_m + \frac{C_{em} - C_m}{J_m} \tag{5}$$

$$\frac{dI}{dt} = -\frac{R}{L}I + \frac{V - E}{L} \tag{6}$$

where :

- C_{em} and C_m are the Electromagnetic and load torque of the motor respectively,
- f represents the friction,
- J_m is the inertia of the motor,
- Ω_m is the velocity of the motor,
- K_E et K_t are constants,
- E is the electromotive force,
- I is the current in the phases of motor,
- V is the tension in the phases of motor,
- L is the inductance of the motor,
- R is the resistance of the motor.

with, $E = K_E\Omega_m$ and $C_{em} = K_t I$.

C. Trajectory Generation

The trajectory generation is part of the control system which imposes setpoints'movement to be followed by moving in a direction towards an end position. Such instructions can be sent directly to the actuators or control loops. The typical movement instructions contain the definition of displacement, kinematic constraints that the robot must respect, the execution time, etc.. So, the movement generation must produce a uniform trajectory without velocity discontinuities. For this, it suffices to impose a rotation angle of the load, it is an angle corresponding to a requested move along a desired speed for an articulated arm.

The problem resides in trajectory calculation generation of the reference position, speed and acceleration instructions which are functions of time and ensure the passage of the robot with a desired path, defined by a sequence of the situations of organ terminal or joint configuration. To obtain the desired displacement, we are interested in the shape of the velocity as a function of time. So, the engine will be stopped when the desired position at the final instant is obtained.

In order to control the movements of the robot, slaving techniques include using information provided by the position sensor. A control law permit to follow the desired speed path which is described as [9] :

$$\Omega_{ref}\ =\ 6\ \frac{(q_f\ -\ q_i)\ t}{t_f^2}\ -\ 6\ \frac{(q_f\ -\ q_i)\ t^2}{t_f^3} \tag{7}$$

where :

- Ω_{ref} is the reference speed,
- q_f is the desired position,
- q_i is the initial position,
- t_f is the time required to achieve the desired position.,
- t is the time,

Similarly, the equations of the the desired position and acceleration can be rewritten as follows:

$$q_{ref}\ =\ q_i\ +\ 3\ \frac{(q_f\ -\ q_i)\ t^2}{t_f^2}\ -\ 2\ \frac{(q_f\ -\ q_i)\ t^3}{t_f^3} \tag{8}$$

$$\gamma_{ref} = 6\frac{(q_f - q_i)}{t_f^2} - 12\frac{(q_f - q_i)\ t}{t_f^3} \quad (9)$$

with :

- q_{ref} is the reference position,
- γ_{ref} is the reference acceleration.

In the following part, we will explain the strategy of PI and sliding mode controllers.

III. PI CONTROLLER

The PID regulator (Proportional, Integral, Derivative) is still widely used in the industrial environment, despite the appearance of other methods of regulation. The regulator is based on a very simple structure whose operation depends only on three coefficients, which are the gain applied to the proportional signal (Kp), integral time constant (Ti) and the derivative time constant (Td)[10]. It allows the cancelation of a static error while allowing the performance of speed higher than a PI controller. The transfer function in the complex domain of Laplace is given by:

$$C(p) = Kp(1 + \frac{1}{Tip} + Tdp) \quad (10)$$

The functional diagram corresponding to the control of flexible joint manipulator by PI controller is illustrated in Figure 2 as follows:

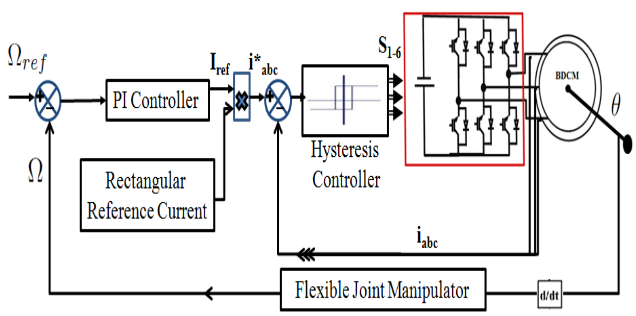

Fig. 2. Diagram of the PI Control block for flexible joint manipulator

IV. SLIDING MODE CONTROLLER

The sliding mode control consists to evolve the sliding surface with a certain dynamic to the point of equilibrium. A sliding surface S is a surface on which the system will follow the desired changes. When the state is maintained on the surface, the system is called sliding regime. Thus, as the sliding conditions are ensured, the dynamics of the system is insensitive to variations in process parameters to modeling errors and some disturbances [11], [12], [13]. The purpose of the sliding mode control is:

* synthesize a surface $S(x) = 0$, such that all trajectories of the system follow a desired behavior tracking, regulation and stability.

* Determine a control law that is able to attract all trajectories of state to the sliding surface $S(x) = 0$ and keep them on the surface.

The design of the sliding mode control relates mainly to determining two steps:

1) The choice of the surface.
2) The determination of the control law.

A. The choice of the sliding surface

Generally, for a system defined by the following equation of state:

$$\dot{x}(t) = f(x,t) + g(x,t)u(t) \quad (11)$$

where:

- x is the state variables vector,
- f is the function describing the evolution of the system over time,
- g is the input function,
- u is the vector control.

The convergence conditions or attractiveness allow dynamic system to converge to the sliding surface and independently remain the disturbance. The Lyapunov function proves the existence condition of sliding surface which is expressed as follows:

$$V = \frac{1}{2}.S^2 > 0 \quad (12)$$

The sufficient condition to guarantee the stability of the system is:

$$\dot{V} = \frac{1}{2}.\frac{\partial}{\partial t}(S^2) \leq -\eta. \mid S \mid < 0 \Longrightarrow S(x).\dot{S}(x) < 0 \quad (13)$$

with: $\eta > 0$.

The equation 12 explains that the square of the distance to the measured surface by S^2 decreases all the time, forcing the trajectory of the system to move towards the surface in both sides.

The choice of sliding surface affects not only the necessary number of these surfaces but also their shape depending on the application and the desired target.

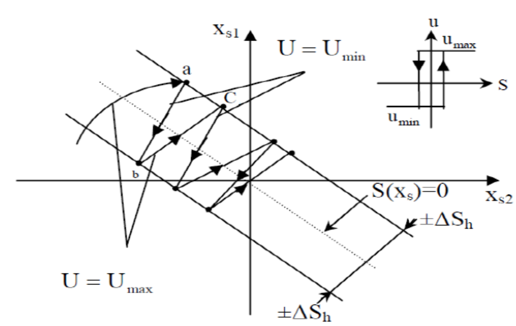

Fig. 3. Sliding phenomenon

The sliding surface is given by the following equation:

$$S = G.x \quad (14)$$

with

$$G = [K_1 \quad 1] \quad (15)$$

$$x = [x_1 \quad x_2]^T = [er_\theta \quad er_\Omega]^T \quad (16)$$

where:

$$er_\theta = \theta_{ref} - \theta \quad (17)$$

$$er_\Omega = \Omega_{ref} - \Omega \quad (18)$$

B. Calculation of the control law

Once the sliding surface is chosen, it remains to determine the necessary control to bring the variable control to the surface and then to the point of equilibrium by maintaining the condition of existence of the sliding mode. One of the essential assumptions in the design of systems with variable structure controlled by sliding mode is that the command should switch between u_{min} and u_{max} instantaneously (infinite frequency), depending on the sign of the sliding surface. In this case, high-frequency oscillations appear in the sliding mode. During sliding mode, the objective is to force the system dynamics to correspond to the sliding surface $S(x)$ by means of the following equation of command:

$$u(t) = u_{eq}(t) + u_s \quad (19)$$

where:

- u: represents the control vector which includes two terms:
- u_{eq}: represents the equivalent part of the sliding mode. It is calculated by knowing the behavior of the system during the description of the sliding model by $\dot{S}(x) = 0$. It is obtained with the conditions of the surface invariance given by equation: $S(x,t) = 0$ and $\dot{S}(x,t) = 0$.
- u_s: represents the discontinuous part. It is determined to guarantee the variable attractiveness for monitoring towards the sliding surface and satisfy the convergence condition. It ensures insensitivity of the system to changes in parameters.

We have also,

$$\dot{S}(x,t) = \frac{dS}{dt} = \frac{\partial S}{\partial x}.\frac{\partial x}{\partial t} = \frac{\partial S}{\partial x}.\dot{x} \quad (20)$$

So,

$$\dot{S}(x,t) = \frac{\partial S}{\partial x}.[f(x,t) + g(x,t)u_{eq}] \quad (21)$$

In sliding mode, the trajectory will remain on the switching surface $S(x) = 0$, ie, its derivative will be null $\dot{S}(x) = 0$ and $u_s = 0$. So,

$$u_{eq}(t) = -[\frac{\partial S}{\partial x}f(x,t)].[\frac{\partial S}{\partial x}g(x,t)]^{-1} \quad (22)$$

The popular solution is to choose u_s as relay. In this case, the command is written as follows:

$$u_s = -K_s.sign(S(x)) \quad (23)$$

where, the positive gain K_s is chosen to satisfy the condition in the equation (23).

The scheme bloc of flexible joint manipulator controlled by sliding mode controller is represented by the figure 4.

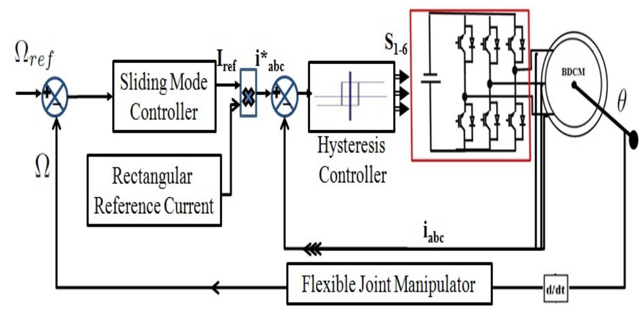

Fig. 4. Diagram of the sliding mode Control block for flexible joint manipulator

V. Simulation Results and Analysis

A. Description of Simulation

The performances of the sliding mode control strategy for flexible joint robot driving by brushless DC motor have been evaluated through simulation works carried out in the MATLAB-SIMULINK environment. Indeed, we adopt the expression of the dynamic model which must be applied to the motor, taking into account the reduction in the yield.

$$C_m = \frac{\Gamma}{\eta N} \quad (24)$$

Our manipulator is a second order system. So, we have a state vector with two variables: $x = [er_\theta \quad er_\Omega]$,
where:

- er_θ is the error of the angular position (rad),
- er_Ω is the error of the angular speed (rpm),

The simulation stage includes two steps:

* In the first step, we have studied the PI controller.

* In the second step, we have studied the sliding mode controller where the sliding surface is described as $S = Gx$ with $x = [er_\theta \quad er_\Omega]$ representing the state vector.

The figures 5, 6, 7, 8 and 9 represent respectively the evolution of speed, position, position error, speed error and electromagnetic torque for the two cases of controllers.

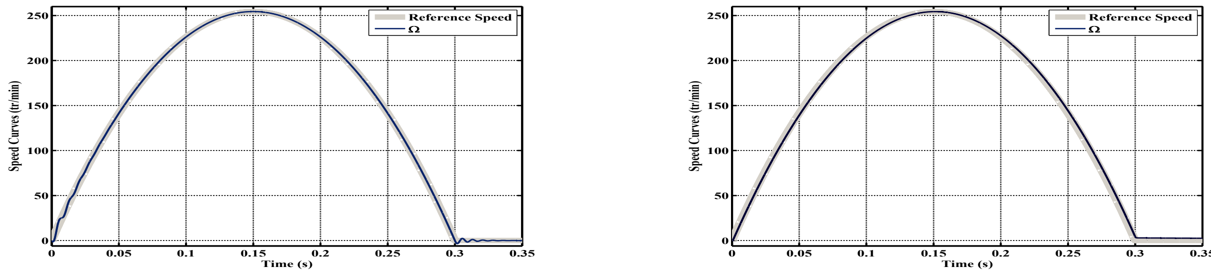

Fig. 5. Evolution curves of the speed, Legend: (left side): PI controller and (right side): Sliding Mode Controller.

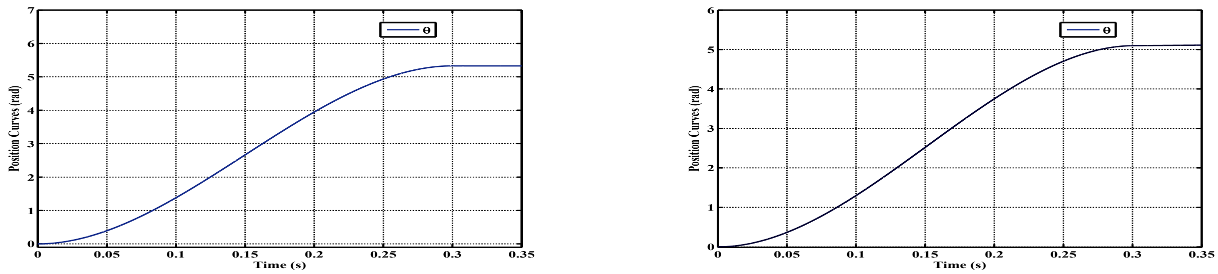

Fig. 6. Evolution curves of the position, Legend: (left side): PI controller and (right side): Sliding Mode Controller.

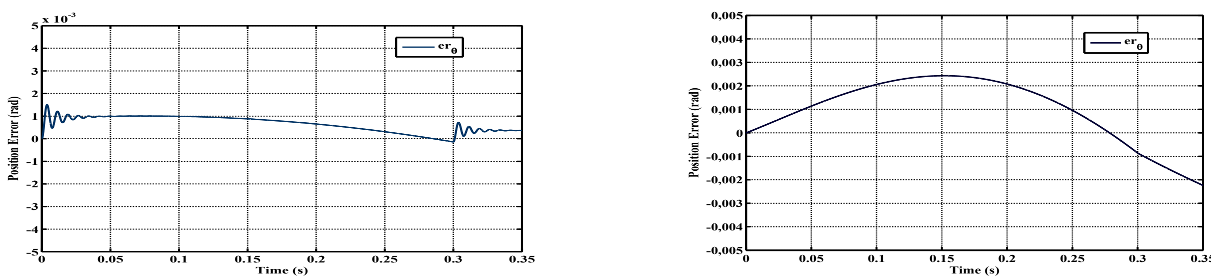

Fig. 7. Evolution curves of the position error, Legend: (left side): PI controller and (right side): Sliding Mode Controller.

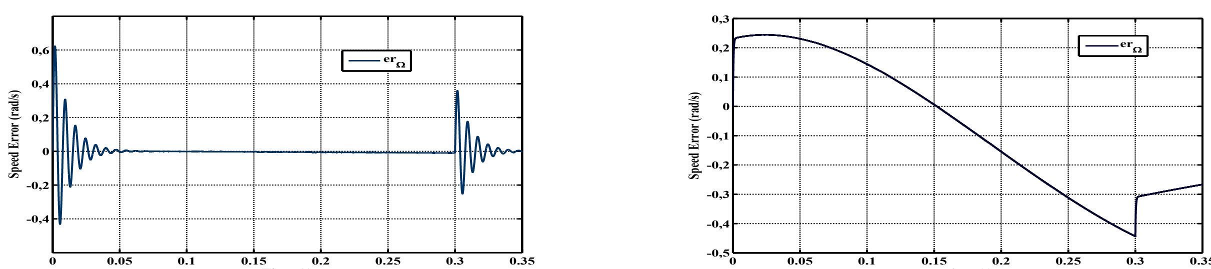

Fig. 8. Evolution curves of the speed error, Legend: (left side): PI controller and (right side): Sliding Mode Controller.

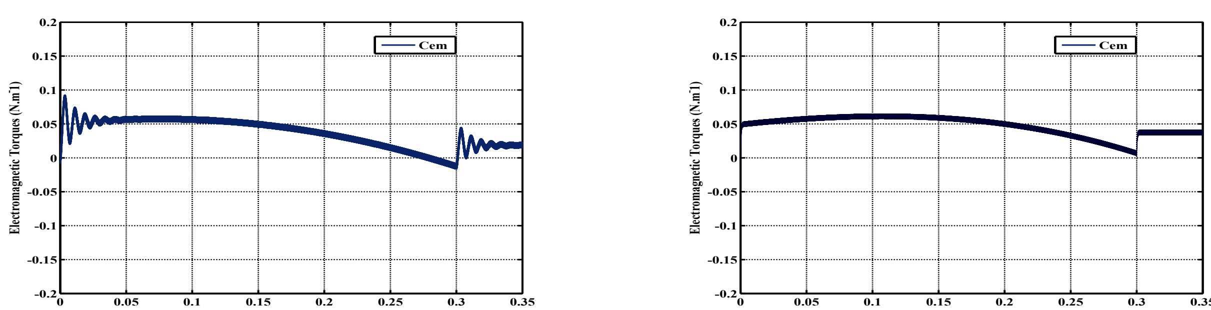

Fig. 9. Evolution curves of the electromagnetic torque, Legend: (left side): PI controller and (right side): Sliding Mode Controller.

B. Analysis

Following the analysis of Figures 5, 6, 7, 8 and 9, it is to be noted that:

- The flexible joint manipulator follows its desired trajectory with a static error near to zero. This confirms the robustness of the PI and sliding mode controller forcing the joint to converge to a desired position with a definite speed for the sliding surface vector choice,

- The simulation shows that the system output for sliding mode controller reaches its desired value with more precision than the PI controller where we observe the presence of the oscillations. Moreover, the errors in positions and speeds are more near to zero in the case of sliding mode controller than PI controller.

The parameters of the system used for simulation are given in table I.

TABLE I. PARAMETERS OF SYSTEM

$R = 0.625\Omega$	$L = 1.595e - 3\ H$	$J_m = 1e - 5$Kg.m^2
$m_1 = 0.8619$Kg	$l_1 = 0.3$m	$J_1 = 0.0065$N.m^2
$N = 74$	$\eta = 0.72$m	f=1.164e-3$Kg.m^2.s^-1$
$K_t = 0.0382$	$K_E = 0.0382$m	

VI. CONCLUSION

A sliding mode control for manipulator driving by brushless DC motor was developed in this paper to guarantee both stability and robustness of the system in presence of joint flexibility. So, the fundamentals of modelization of flexible joint manipulator and BDCM motor were firstly recalled. Then, a PI controller was designed in a second step. A sliding mode control strategy was proposed in a third step by giving both of sliding surface and control law. Finally and in a fourth step, the PI and sliding mode control strategies considering a sliding surface vector with two state variable vector: $x = [er_\theta \quad er_\Omega]$ previously described have been implemented in matlab simulink environment. Simulation results have shown that those control strategies lead to high performance. The simulation results have shown also that the reduction of oscillations is influenced by the sliding mode controller.

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