10th International conference on Sciences and Techniques
of Automatic control & computer engineering
December 20-22, 2009, Hammamet, Tunisia

Non-Linear signal filtering with empirical mode decomposition

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Abstract. *This paper presents a new technique called Empirical Mode Decomposition (EMD) applied to filter a stationary and non-stationary signal and demonstrates the influence of the number of filtered IMFs on the SNR. The result of comparaison with wavelet transforms technique demonstrates the performance of the proposed approach. Noisy signal is decomposed adaptively into oscillatory components called Intrinsic Mode Functions (IMFs) by means of a process called sifting. The EMD denoising involves filtering or thresholding each IMF and reconstructs the estimated signal using the processed IMFs. In this paper, we apply this approach to denoise the stationary and non-stationary signals to achieve the highest SNR as the number of IMFs filtered..We demonstrate how the proposed method improves the interpretive information of the signal by com paring it with widely used DWT denoising schemes.*

Keywords. *Empirical mode decomposition, Signal denoising, SNR, IMF, wavelet transform.*

1. Introduction

Signal filtering and noise reduction are fundamental problems in various applications includes medical signal, analysis and speech signal processing [1].

Generally, the signals are noisy when they were purchased. Filtering or denoising is often required to improve the quality of these signals and the power later use. Filtering or denoising are often required to improve the quality of these signals and classical linear methods such as Wiener filter, the averaging or Gaussian are the most used because of their simplicity and ease of implementation [2]. The improvement of this method is demonstrated in the case of stationary and non stationary signals.

In recent years, firstly wavelet transform technique is applied to analyze the non linear and the non stationary signals then the application of Empirical Mode Decomposition (EMD) technique is applied to analyze nonlinear and non-stationary signals has gained importance [3] .

The paper is organized as follows. In section 2, the EMD methods used in this paper is shortly described. Next in section 3 we describe this method in signal denoising and filtering. Further are experiment results on the EMD approach applied in stationary and non stationary signal in section 4. Finally, a conclusion is presented in section 5.

2. Empirical Mode Decomposition

Empirical mode decomposition proposed by Huang in 1998 deals with nonlinear and non-stationary signals [4]. The EMD decomposes a signal into a collection of oscillatory modes, Called intrinsic mode functions (IMF), which represent fast to slow oscillations in the signal with correspond to high frequency (or detail in wavelet terminology) and low frequency (approximation). In IMFs the frequency bands are different to each other, depends on frequency in real signal.

An IMF is defined as a function with equal number of extrema and zero crossings (or at most differed by one) with its envelopes, as defined by all the local maxima and minima, being symmetric with respect to zero. [5]

Empirical mode decomposition (EMD) adaptively decomposes a multicomponent signal x (t) into M Intrinsic Mode Functions (IMFs).

The sifting procedure to obtain IMFs of the signal x (t) is described as follows [6].

1) Identify all the maxima and the minima in the signal x (t).
2) Generate its upper and lower envelopes using cubic spline interpolation.
3) Compute the point by point local mean m_1 from upper and lower envelopes.
4) Extract the details, $h_1 = x(t) - m_1$.
5) Check the properties of h_1 and iterate k times, then $h_1(k) = h_1(k-1) - m_1(k)$ becomes the IMF once it satisfies some stopping criterion. It is designated as first IMF $c_1 = h_1(k)$.
6) Repeat steps 1) to 5) on the extracted data $r_1(k) = x(t) - c_1$.
7) The step 6) is repeated until all the IMFs and residual is obtained.

The result of the sifting procedure is that x (t) will be decomposed into $IMF_j(t)$, j = 1. . . N and a residual $r_N(t)$:

$$x(t) = \sum_{j=1}^{N} IMF_j(t) + r_N(t) \tag{1}$$

The EMD approach can be applied to denoise signals or images as well as wavelet technique.

3. Denoising by EMD

This approach is based on the reconstruction of filtered signal by filtering all IMFs previously pre-treated. The method is seen as a technique for denoising. The idea of EMD is to Threshold (as defined by wavelet denoising) each IMF separately [7].

Thus, if we consider $f_j(t)$ a non noisy finite length T and its noisy version $IMF_j(t)$ by assumed white Gaussian and additive noise, $b_j(t)$ then

$$IMF_j(t) = f_j(t) + b_j(t);\ \ j=1,\ldots,N \tag{2}$$

We then define $\tilde{f}_j(t)$ an estimate of f_j (t) based on the observation of noisy $IMF_j(t)$. The denoising signal (reconstructed) $\tilde{x}_j(t)$ is given by the relation:

$$\tilde{x}(t) = \sum_{j=1}^{N} \tilde{f}_j(t) + r_N(t);\ \ j=1,\ldots,N \tag{3}$$

$\tilde{f}_j(t)$ is obtained by a hard or soft thresholding of the decomposed IMF.

3.1. Hard thresholding

The hard thresholding is an intuitive method to keep only those samples whose amplitudes are higher to τj and replacing others with zero [9].

$$\tilde{f}_j(t) = \begin{cases} IMF_j(t) & \text{if } \left|IMF_j(t)\right| > \tau_j \\ 0 & \text{if } \left|IMF_j(t)\right| \le \tau_j \end{cases} \tag{4}$$

τ_j is called universal threshold of Donoho described by equation (5)[9].

$$\tau_j = \tilde{\sigma}_j \sqrt{2.\ln(T)} \tag{5}$$

$\tilde{\sigma}_j$ is also called the noise level of the j^{th} IMF and T is the number of samples.

$$\tilde{\sigma}_j = \mathrm{MAD}_j / 0.6745 \qquad (6)$$

$$\mathrm{MAD}_j = \mathrm{Median}\left\{\left|\mathrm{IMF}_j(t) - \mathrm{Median}\left\{\mathrm{IMF}_j(t')\right\}\right|\right\} \qquad j = \{1, \ldots, N\} \qquad (7)$$

Where MAD is the Median Absolute Deviation and Median { } is the median of the variable.

3.2. Soft thresholding

Soft thresholding is less excessive than the hard thresholding and can reduce the noise of each IMF. This method of mitigation τ_j decreases the amplitude of all noisy samples ($\mathrm{IMF}_j(t)$ values) that are above the threshold τ_j. The estimate of denoised versions $\tilde{f}_j(t)$ $\mathrm{IMF}_j(t)$ is associated calculated as following [8]:

$$\tilde{f}_j(t) = \begin{cases} \mathrm{IMF}_j(t) - \tau_j & \text{if } \mathrm{IMF}_j(t) \geq \tau_j \\ 0 & \text{if } \left|\mathrm{IMF}_j(t)\right| < \tau \\ \mathrm{IMF}_j(t) + \tau & \text{if } \mathrm{IMF}_j(t) \leq -\tau \end{cases} \qquad (8)$$

The soft thresholding leads to smoother estimates than those obtained by hard thresholding approach [9].

4. Experimental results

We applied the approach to the EMD denoising non-stationary signals. The noise used is an additive Gaussian noise with gamma $\delta = 50$.

The EMD approach adaptively decomposes the noisy signal into a set of intrinsic mode function (IMFs) and a residual one. The result of decomposition is illustrated in figure1.

To implement the EMD approach, described in paragraph 2, on the noisy non-stationary signal and find the reconstructed signal, we add all IMFs filtered by the equation (3).

Figure 2 and 3 present respectively an example of stationary and non-stationary signal denoised with the EMD approach.

It was noted that there is no need to filter all IMFs decomposed a noisy signal but just filter the first IMFs, which contains the most noise on this example the first 3 IMFs are filtered and gives maximum of the SNR (36 db). So the Figure 4 explains the behavior of SNR on the number of IMFs filtered.

We are compared the EMD denoising approach with DWT technique. The Figure 5 shows the difference between the EMD denoising approach and the DWT applied on a non-stationary signal.

The result of the SNR level is 37db for the EMD and 14 for DWT technique denoising approach when 3 IMFs are filtered. This shows these values of SNR demonstrated that this approach (EMD) applied on non-stationary signals has shown the efficiency in SNR criteria compared to other approaches used as the DWT approach, especially for the Gaussian additive noise.

It was noted that the EMD for non-stationary signal denoising is more effective for Gaussian noise than other used filters.

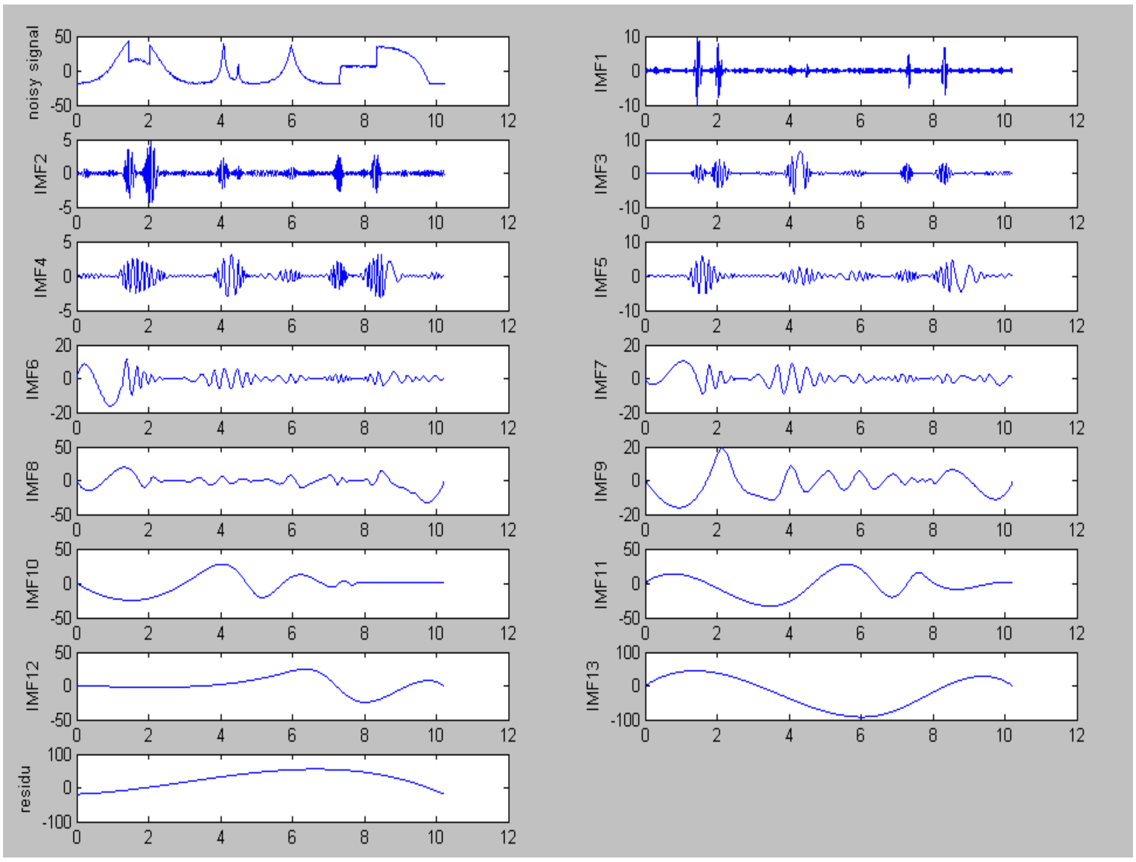

Fig. 1. Empirical mode decomposition of a noisy non-stationary signal.

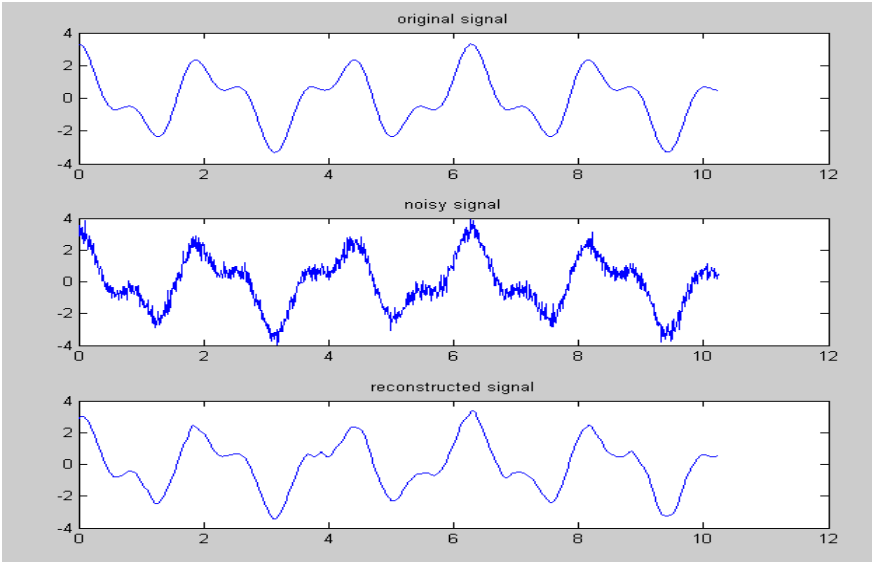

Fig 2. Results of denoising by EMD approach for the stationary signal
x(t)=cos(7 t)+2 cos(3 t)+0.3 cos(t)

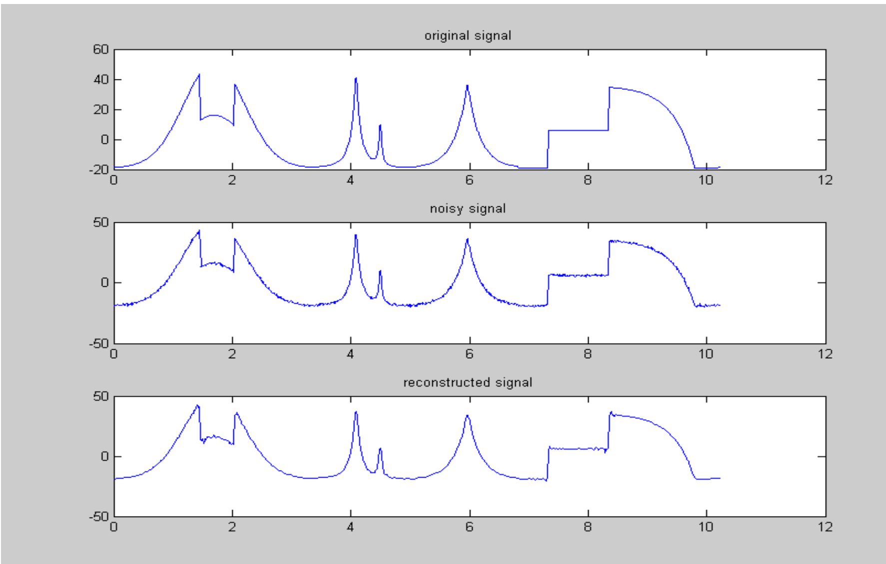

Fig 3. Results of denoising by EMD approach for the non-stationary signal.

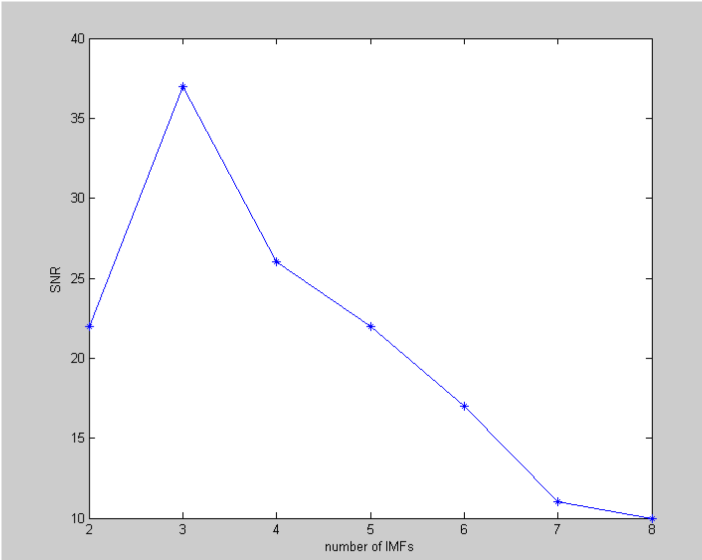

Fig.4. The behavior of SNR as a function of IMFs filtered.

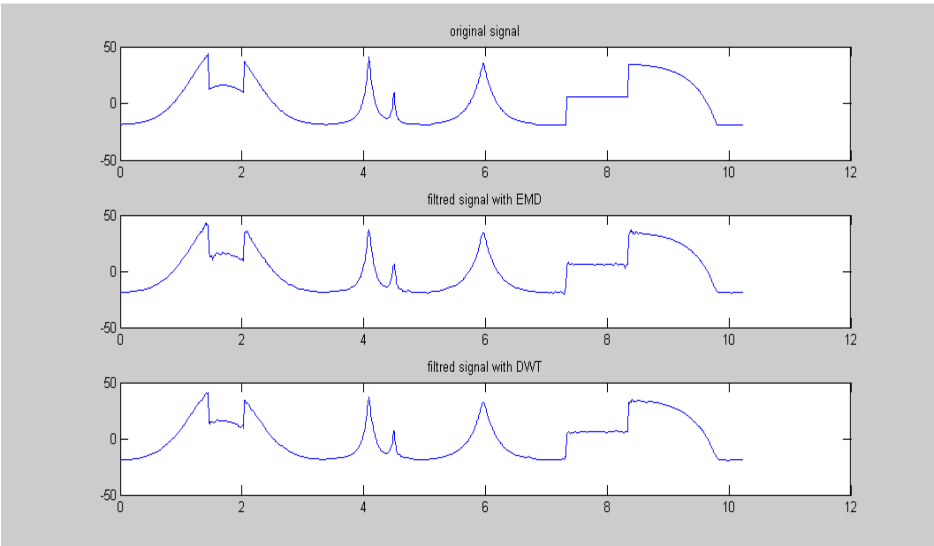

Fig.5. Comparison of the EMD method with DWT for denoising non-stationary signals.

5. Conclusions

An efficient technique for signal denoising using Empirical Mode Decomposition (EMD) has been proposed. This contribution consists on applying the EMD approach for the denoising the non-stationary signals. Original signal is decomposed into successive IMFs using empirical mode decomposition. In the next step, Each IMF is filtered separately by soft thresholding method. The reconstruction of the signal is performed by adding the different filtered IMFs. During implementation of this approach it was noted that there is no need to filter all IMFs components of the original signal; just the first ones (two or three) are necessary to give a good result. The simulations are performed for stationary and non stationary signals. The application of this approach in the denoising field gives a better performance compared with Discrete Wavelet technique.

6. References

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